ON THE DESIGN OF THERMOTECHNICAL EXPERIMENTS

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The advantages are discussed of applying the modern apparatus of experiment design to the solution of problems in thermotechnical and thermophysical measurements. The determination of the heat transfer coefficient in film-flow evaporators is used as an illustration to show how the number of tests can be significantly reduced by a factorial design of experiments.

One purpose of thermotechnical and thermophysical experiments is to obtain mathematical relations between a given studied quantity and the factors affecting it. This relation is often adequately well described by power expressions [1, 2]:

$$y = Ax_1^{n_1}X_2^{n_2}\ldots x_k^{n_k}, \tag{1}$$

where y is the studied quantity, x_1, x_2, ..., x_k are dimensional or dimensionless (universal) determining quantities (factors), and k is the number of such factors.

In this article the authors consider the feasibility and the technique of experiment design which would ensure a significant reduction in the number of tests for determining n_1, n_2, ..., n_k and A in Eq. (1).

Schemes have been developed in the theory of experiment design which would minimize the number of tests for determining the sought relations expressable in the form of linear equations, power laws, or nth-degree polynomials [3-5]. An experiment for the determination of A, n_1, n_2, ..., n_k in Eq. (1) can be implemented according to the designed total factorial experiment. For each factor the number of its values to be measured can be reduced to two, since Eq. (1) can be linearized by taking logarithms and changing to variables:

$$\begin{aligned} z = \lg y &= \lg A + n_1 \lg x_1 + n_2 \lg x_2 + \ldots + n_k \lg x_k \\ &= z_0 + n_1 z_1 + n_2 z_2 + \ldots + n_k z_k, \end{aligned} \tag{2}$$

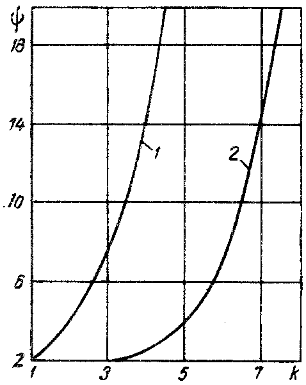

Fig. 1. Reduction of the number of tests, with an increasing number of factors, by means of a total factorial design (curve 1) and by means of a partial factorial design (curve 2) of the experiment.

and, in order to determine the coefficients in this equation, it is sufficient to perform an experiment at two factor levels. In this case one designs an experiment for determining n_1, n_2, ..., n_k and $z_0 = \log A$, with the quantities $z = \log x$ representing the factors. The number of tests necessary for realizing all possible combinations of factor levels is here

$$N_1 = 2^k. \tag{3}$$

The reduction in the number of tests attainable by such an experiment design is estimated by the quantity

$$\psi = \frac{N_2}{N_1}, \tag{4}$$

where $N_2 = m_{x1}m_{x2}\ldots m_{xk}$ is the number of tests performed once according to the conventional plan, with m_{xi} denoting the number of measurements for each factor. When $m_{x1} = m_{x2} = \ldots = m_{xk} = m - N_2 = m^k$ and on the basis of (4), we have

$$\psi = \left(\frac{m}{2}\right)^k. \tag{5}$$

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TABLE 1. Design Matrix

Test No.	Factor level				
	to natural scale		in code		
	Δt, °C	Γ, kg/msec	z_0	z_1	z_2
1	22	2,85	+1	+1	+1
2	2	0,65	+1	−1	−1
3	22	2,85	+1	−1	+1
4	2	0,65	+1	+1	−1

Calculated values of ψ according to various plans are shown in Fig. 1. These data (curve 1) indicate a significant reduction of the number of tests in a total factorial experiment, as compared to the number of tests in a conventional experiment where the value of each one factor is varied with all other factors held constant and, subsequently, both the constant and the variable factors are changed. Curve 2 indicates the reduction of the number of tests in a partial factorial experiment [5], such an experiment becoming advantageous when $k > 3$.

The procedure for designing an experiment with a significant reduction of the number of tests was followed by the authors in a determination of the heat transfer coefficient in film-flow evaporators [6].

The purpose of such an experiment was to establish the relation between the heat transfer coefficient and its determining factors in the form

$$\alpha = A\Delta t^m \Gamma^n. \tag{6}$$

A measurement of quantities A, m, and n by the conventional method requires at least 16 tests in a single experiment, on the assumption that each factor will be varied through four levels. In a total factorial experiment, on the other hand, the number of tests for measuring A, m, and n in Eq. (6) is found from (3) to be only four. The conditions of the experiment are put in the form of a design matrix in Table 1. The values of the parameters in Eq. (6) are found, by the method of least squares, to be $A = 11{,}750$, $m = -0.63$, and $n = 0.22$.

The adequacy of the resulting equation is checked on the basis of Fisher's criterion [5]:

$$F = \frac{S^2_{ad}}{S^2_y} \tag{7}$$

with the dispersion of adequacy S^2_{ad} indicating the discrepancy between measured and calculated values of α and with the dispersion S_y defining the variability of recurrent tests.

For our experiment the F-criterion, which depends on the number of recurrent tests and on the confidence interval, was found to be equal to 2.4 according to the Tables in [5]. The F_p-criterion calculated according to (7) was found equal to 1.5 and thus smaller than its tabulated value, indicating a satisfactory agreement between calculation and measurement.

Thus, the heat transfer coefficient in film-flow evaporators and the data in Fig. 1 illustrate the advantages of applying the modern techniques of experiment design to heat transfer problems.

The equations which describe thermotechnical and thermophysical experiments may often differ from Eq. (1). In that case, 2^k-designs are applicable to the initial stage of experimentation only and special designs, such as those described in [3-5] are adopted later on.

NOTATION

y	is the quantity under study;
$x_1, x_2, \ldots, x_k$	are the determining factors;
k	is the number of factors;
m	is the number of measurements of each factor;
N	is the number of tests;
α	is the heat transfer coefficient, $W/m^2 \cdot °C$;
Δt	is the temperature difference between wall and evaporating film, °C;
Γ	is the trickle rate, kg/msec;
F	is the Fisher criterion.

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